The Synchronization Problem for a Class of Supply Chain Complex Networks

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Abstract— **In this paper, we investigate the global adaptive synchronization problem for a class of supply chain complex networks that have nonlinearly coupled identical nodes and an asymmetrical coupling matrix. We derive, in particular, a sufficient condition for global synchronization when the coupling strength is not excessively large by applying a suitably chosen adaptive controller and demonstrate the effectiveness of this theory by numerical examples.**

Index Terms— **Supply chain, complex networks, synchronization, adaptive, nonlinearly coupled, asymmetrical coupling.**

I. Introduction

There is a great interest in the synchronization of complex networks in recent years from disciplines as diverse as the mathematical, physical, biological and physiological sciences because of the many potential applications of the phenomenon (see [1], [2] and the references therein). Loosely speaking, synchronization is the process in which two or more dynamical systems adjust a given property of their motion to a common behavior in the limit of infinite time due to coupling or forcing [3]. Complex networks that are synchronized, in particular, have many important real-world applications such as in the enhancement of communications security, in seismology and in parallel image processing, among others [4]- [9]. The synchronization phenomenon has already been much investigated. The properties of the invariant manifold, for example, have been used to describe the synchronization process in [10] and the synchronization problems and boundedness of linearly coupled oscillators have been considered in [11] by using the semi-passivity property. [12] studied local synchronization by introducing a master stability function that is based on the transverse Lyapunov exponents and [13] showed how a coupled complex network can be pinned to a homogenous solution by using a single controller and proposed an effective approach to adapt the coupling strength. [14] constructed a novel coupling scheme with cooperative and competitive weight couplings to stabilize arbitrarily selected cluster synchronization patterns in connected networks with identical nodes while [15] studied the cluster synchronization of dynamical networks with community structure and nonidentical nodes in the presence or absence of time delays by feedback control.

A supply chain is a complex network of human agents (such as manufacturers, retailers and consumers) that interact with each other by completing business transactions and supply chain analysis is therefore an interdisciplinary science that spans the subject areas of manufacturing, transportation, logistics and retailing/marketing (see [16] and the references therein). Two supply chains are synchronized if the agents could all be coerced to operate in a mutually supportive and seamless manner and supply chain synchronization usually starts by ensuring that every agent knows the exact tasks to be performed (e.g. storing goods, price marking), the time-frame (e.g. lead times and deadlines) and the way (e.g. to what operating specifications) in which to perform those tasks as well as the results to be expected (e.g. sales quotas, customer satisfaction ratings).

Most synchronization studies that have been conducted so far, however, have been focussed on oscillators under linear coupling or bidirectional nonlinear coupling [13], [17], [18] and [19]. This is unsatisfactory because unidirectional communication is prevalent in practical applications such as in radio and television broadcasting as well as in other forms of sensed information flow that are typical in schooling and flocking phenomena [20]. A more detailed analysis of unidirectional communication is thus in order.

In this paper, we consider complex networks that are made up of N identical nonlinearly and diffusively coupled nodes in which every node is an n-dimensional dynamical system of a supply chain model. The state equations of this network are

$$\dot{x}^i(t) = f(x^i(t)) - \sigma \sum_{i=1}^{N} l_{ij} h(x^j(t)), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $\sigma > 0$ is the interaction strength between the various business entities (e.g. manufacturers, customers. etc.) $x^i(t)$ and σ is the coupling strength. $x^i(t)$ is n dimensional due to a number of features relating to the

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business entities in global supply chains such as the exchange rates, the corporate tax rates, the tariffs, and the direct export incentives. $f(x)$ is the behavior function of $x^i(t)$ and the diffusive coupling matrix $L = [l_{ij}] \in R^{N\times N}$ represents the interactions between the different $x^i(t)$s where $l_{ij} \le 0$, $i \ne j$ and $l_{ii} = \sum_{j=1,i\neq j}^{N} l_{ij}$, and can be regarded as the graph Laplacian when the coupled complex network is viewed as a weighted directed graph. If the nodes i and j are connected, then $l_{ij} \ne 0$; otherwise $l_{ij} = 0$ and the diagonal elements of the coupling matrix L are $l_{ii} = -\sum_{i=1}^{N} l_{ij} = -k_i$, $i = 1, 2, ..., N$, where k_i denotes the degree of node i. The nonlinear coupling function $h(\cdot) : R^n \to R^n$ could be regarded as the interaction function of $x^i(t)$, which is continuous and has the form $h(x^i(t)) = (h_1(x^i_1(t)), \ldots, h_n(x^i_n(t)))^T$, $i = 1, 2, \ldots, N$. Other criteria for global synchronization include the projecting of the nonlinear coupling function onto a linear one with the difference between the two being a disturbing function. The theoretical values of the coupling strengths that satisfy these conditions, however, are usually much larger than are needed in practice [21] and creates an issue that has to be addressed. In this paper, we investigate the adaptive synchronization of nonlinearly coupled directional complex networks where we nontrivially extend the work of [20] in which the coupling strength is constant. In particular, we derive one sufficient criterion for global synchronization by constructing an adaptive coupling strength controller

An outline of this paper is as follows. In Section II, some necessary definitions and lemmas are given. In Section III, a sufficient condition for the global adaptive synchronization of nonlinearly-coupled systems is derived. In Section IV, numerical examples are presented to show the validity of the theoretical analysis. Finally, we conclude this paper in Section V.

II. Preliminaries

In this section, we present some lemmas and assumptions that are required for this paper.

Definition 1: [20] A matrix $L = (l_{ij})_{i,j=1}^{N}$ is said to belong to class $\mathbf{A_1}$ and written as $L \in \mathbf{A_1}$ if

1) $l_{ij} \ge 0$, $i \ne j$, $l_{ii} = \sum^N j = 1, j \ne il_{ij}$, $i = 1, 2, ..., N$
2) L is irreducible.

If $L \in \mathbf{A_1}$ is symmetrical, then we say that L belongs to class $\mathbf{A_2}$ and write $L \in \mathbf{A_2}$.

Lemma 1: [20] If $L \in A_1$, then $rank(L) = N - 1$, i.e. 0 is an eigenvalue of L of multiplicity 1 and all the nonzero eigenvalues of L have positive real parts.

Lemma 2: [20] If $L \in \mathbf{A_1}$, then

1) $\mathbf{1} = (1, 1, ..., 1)^T$ is a right eigenvector of L corresponding to the eigenvalue 0 of multiplicity 1, i.e. $A \cdot \mathbf{1} = 0$;
2) If $\xi = (\xi^1, \xi^2, ..., \xi^N)^T$ is a left eigenvector of L corresponding to the eigenvalue 0, i.e. $\xi^T L = 0$, then $\xi^i > 0$, $i = 1, 2, ..., N$ with multiplicity 1. We shall assume that $\sum_{i=1}^{N} \xi^i = 1$ throughout this paper.

Definition 2: [13] If there are positive matrices $P = \mathrm{diag}\{p_1, p_2, \ldots, p_n\}$, $\Delta = \mathrm{diag}\{\delta_1, \delta_2, \ldots, \delta_n\}$, then we say that $f(x, t) \in QUAD(P, \Delta, \eta)$ if f satisfies the following inequality:

$$(x-y)^T P\big((f(x,t) - f(y,t)) - \Delta(x-y)\big) \le -\eta(x-y)^T(x-y),$$

for some $\eta > 0$, $x, y \in R^n$ and $t > 0$.

Indeed, the class $QUAD(P, \Delta, \eta)$ contains many of the benchmark chaotic systems such as the Lorenz system, the Chen system, the Lü system and the unified chaotic system.

Definition 3: [20] A nonlinear function $g(x) : R \to R$ is said to belong to the acceptable nonlinear coupling function class and written as $g \in NCF(\gamma, \beta)$ if there exist two nonnegative scalers γ and β such that $g(\omega) - \gamma\omega$ satisfies the Lipschitz condition

$$|g(\omega_1) - g(\omega_2) - \gamma(\omega_1 - \omega_2)| \le \beta|\omega_1 - \omega_2|$$

for any $\omega_1, \omega_2 \in R$.

Definition 4: [20] Let $\xi = (\xi^1, \xi^2, \ldots, \xi^N)^T$ be a left eigenvector of L corresponding to the eigenvalue 0, i.e. $\xi^T L = 0$ and let $\Xi = \mathrm{diag}\{\xi\}$, $I_n = \mathrm{diag}\{\mathbf{1}_n\}$. If we define $U = \Xi - \xi\xi^T$, $Q = I_n - \frac{1}{N}\mathbf{1}_n \cdot \mathbf{1}_n^T$

Then it is clear that $-Q \in \mathbf{A_2}$ and that if $M \in R^{m\times n}$ is a zero-row-sum matrix, then $MQ = M$. Thus, we have

$$x^T My = x^T MQy \le \frac{1}{2}\left(x^T MM^T x + y^T Qy\right). \quad (2)$$

Lemma 3: [13] If $L = [l_{ij}] \in \mathbf{A_2}$, then for any two vectors $u = [u_1, u_2, \ldots, u_n]^T$ and $v = [v_1, v_2, \ldots, v_n]^T$, we have

$$u^T Lv = \sum_{i=1}^{n}\sum_{j=1}^{n} u_i l_{ij} v_j = -\sum_{j>i} a_{ij}(u_i - u_j)(v_i - v_j).$$

III. Main Result

In this paper, we consider network functions with coupling strengths that vary with time and aim to determine the appropriate coupling strength adaptive laws that will enable the system to attain complete global synchronization. More precisely, complete global synchronization for the system (1) can be achieved by writing it as

$$\dot{x}^i(t) = f(x^i(t)) - \sigma(t)\sum_{i=1}^{N} l_{ij} h(x^j(t)), \quad i = 1, 2, \ldots, N, \quad (3)$$

for some adaptive coupling strength controller $\sigma(t)$ and by selecting a suitable adaptive law $\dot{\sigma}(t)$ so that $||x^i(t) - x^j(t)|| = 0$ as $t \to \infty$ for $i, j = 1, 2, ..., N$.

Theorem 1: Let $f(\cdot) \in QUAD(P, \Delta, \varepsilon)$, $L = [l_{ij}] \in \mathbf{A_1}$ and $h_k \in NCF(\gamma_k, \beta_k)$ for some $\beta_k > 0$, $k = 1, 2, \ldots, \varsigma$ and $\beta_k = 0$ for $k = \varsigma + 1, \varsigma + 2, \ldots, n$. Also let the adaptive coupling strength $\sigma(t)$ satisfy

$$\dot{\sigma}(t) = \alpha\sum_{i=1}^{N} \xi^i (x^i(t) - x^{\xi}(t))^T P(x^i(t) - x^{\xi}(t)), \quad (4)$$

for some adaptive law $\alpha > 0$ and initial values $\sigma(0) \geq 0$. If there exist positive scalers θ_k, $k = 1, 2, ..., \varsigma$, such that the inequality

$$-\gamma_k\{\Xi L\}^s + \frac{\theta_k}{2}\Xi LL^T\Xi + \frac{\beta_k^2}{2\theta_k}Q \leq 0; \quad k = 1, 2, \dots, \varsigma \tag{5}$$

holds, where $\{\Xi L\}^s = \{\Xi L + L^T\Xi\}/2$, then the nonlinear coupling system (3) can attain global synchronization.

Proof: Let $X(t) = (x^1(t)^T, x^2(t)^T, \dots, x^N(t)^T)^T$; $F(X(t)) = (f(x^1(t))^T, f(x^2(t))^T, \dots, f(x^N(t))^T)^T$; $H(X(t)) = (h(x^1(t))^T, h(x^2(t))^T, \dots, h(x^N(t))^T)^T$; $\mathbf{L} = L \otimes P$. Then system (3) can be written as

$$X(t) = F(X(t)) - \sigma(t)\mathbf{L}H(X(t)). \tag{6}$$

Now let $x^\xi(t) = \sum_{i=1}^N \xi^i x^i(t)$ and choose a constant $\alpha > 0$ and a Lyapunov function

$$\begin{aligned}V(X(t)) =&\frac{1}{2}\sum_{i=1}^N \xi^i(x^i(t) - x^\xi(t))^T P(x^i(t) - x^\xi(t))\\ &+ \frac{1}{2\alpha\rho}(\sigma - \rho\sigma(t))^2\\ =&\frac{1}{2}X(t)^T\mathbf{U_P}X(t) + \frac{1}{2\alpha\rho}(\sigma - \rho\sigma(t))^2.\end{aligned} \tag{7}$$

where $\mathbf{U_P} = U \otimes P$.

Differentiate the function $V(X(t))$ along the system (3) under the control (4) and let $\mathbf{\Delta} = I_N \otimes \Delta$. Then we have

$$\begin{aligned}\dot{V}(t) =&X(t)^T\mathbf{U_P}\big(F(X(t)) - \sigma(t)\mathbf{L}H(X(t))\big)\\ &- \sigma X(t)^T\mathbf{U_P}X(t) + \rho\sigma(t)X(t)^T\mathbf{U_P}X(t)\\ =&X(t)^T\mathbf{U_P}\big(F(X(t)) - \mathbf{\Delta}X(t)\big)\\ &+ \Big[X(t)^T\mathbf{U_P}\mathbf{\Delta}X(t) - \sigma(t)X(t)^T\mathbf{U_P}\mathbf{L}H(X(t))\Big]\\ &- \Big[\sigma X(t)^T\mathbf{U_P}X(t) + \rho\sigma(t)X(t)^T\mathbf{U_P}X(t)\Big]\\ =&V_1(t) + V_2(t) + V_3(t)\end{aligned} \tag{8}$$

and by noting that $f(\cdot) \in QUAD(P, \Delta, \varepsilon)$, $U \in \mathbf{A_2}$ and using Lemma 3, we have

$$\begin{aligned}V_1(t) =&X(t)^T\mathbf{U_P}\big(F(X(t)) - \mathbf{\Delta}X(t)\big)\\ =&-\sum_{i>j}^N u_{ij}\big(x^i(t) - s^j(t)\big)^T P\big(f(x^i(t)) - f(x^j(t))\\ &- \Delta(x^i(t) - x^j(t))\big)\\ \leq&-\varepsilon\sum_{i>j}^N u_{ij}\big(x^i(t) - s^j(t)\big)^T\big(x^i(t) - s^j(t)\big)\\ =&-\varepsilon X(t)^T U \otimes IX(t)\end{aligned} \tag{9}$$

Let $\tilde{x}_k(t) = (x_k^1(t), x_k^2(t), \dots, x_k^N(t))^T$, $\tilde{h}_k(\tilde{x}_k(t)) = (h_k(x_k^1(t)), h_k(x_k^2(t)), \dots, h_k(x_k^N(t)))^T$ for $k = 1, 2, \dots, n$. Since $UL = \Xi L$, which has zero-row-sum, and $h_k \in NCF(\gamma_k, 0)$ for $k = \varsigma + 1, \dots, n$, we have

$$\sum_{k=\varsigma+1}^n p_k\tilde{x}_k^T(t)\Xi L(\tilde{h}_k(\tilde{x}_k(t)) - \gamma_k\tilde{x}_k(t)) = 0.$$

Hence,

$$\begin{aligned}V_2(t) =&X(t)^T\mathbf{U_P}\mathbf{\Delta}X(t) - \sigma(t)X(t)^T\mathbf{U_P}\mathbf{L}H(X(t))\\ =&\sum_{k=1}^n p_k\delta_k\tilde{x}_k(t)^T U\tilde{x}_k(t)\\ &- \sigma(t)\sum_{k=1}^n p_k\tilde{x}_k(t)^T\Xi L\tilde{h}_k(\tilde{x}_k(t))\\ =&\sum_{k=1}^n p_k\tilde{x}_k(t)^T\big(\delta_k U - \sigma(t)\gamma_k\Xi L\big)\tilde{x}_k(t)\\ &- \sigma(t)\sum_{k=1}^n p_k\tilde{x}_k(t)^T\Xi L\big(\tilde{h}_k(\tilde{x}_k(t)) - \gamma_k\tilde{x}_k(t)\big)\\ =&\sum_{k=1}^n p_k\tilde{x}_k(t)^T\big(\delta_k U - \sigma(t)\gamma_k\Xi L\big)\tilde{x}_k(t)\\ &- \sigma(t)\sum_{k=1}^\varsigma p_k\tilde{x}_k(t)^T\Xi L\big(\tilde{h}_k(\tilde{x}_k(t)) - \gamma_k\tilde{x}_k(t)\big)\end{aligned} \tag{10}$$

and since $\Xi L \in \mathbf{A_1}$ and $h_k \in NCF(\gamma_k, \beta_k)$, it follows from Lemma 3 that

$$\begin{aligned}&\sum_{k=1}^\varsigma p_k\tilde{x}_k(t)^T\Xi L\big(\tilde{h}_k(\tilde{x}_k(t)) - \gamma_k\tilde{x}_k(t)\big)\\ =&\sum_{k=1}^\varsigma p_k\tilde{x}_k(t)^T\Xi LQ\big(\tilde{h}_k(\tilde{x}_k(t)) - \gamma_k\tilde{x}_k(t)\big)\\ \leq&\frac{1}{2}\sum_{k=1}^\varsigma p_k\Big(\theta_k\tilde{x}_k(t)^T\Xi LL^T\Xi\tilde{x}_k(t)\\ &+ \frac{1}{\theta_k}\big(\tilde{h}_k(\tilde{x}_k(t)) - \gamma_k\tilde{x}_k(t)\big)^T\\ &\times Q\big(\tilde{h}_k(\tilde{x}_k(t)) - \gamma_k\tilde{x}_k(t)\big)\Big)\\ =&\frac{1}{2}\sum_{k=1}^\varsigma p_k\Big(\theta_k\tilde{x}_k(t)^T\Xi LL^T\Xi\tilde{x}_k(t)\\ &- \frac{1}{\theta_k}\sum_{j>i}^\varsigma Q_{ij}\big(h_k(x_k^j(t)) - \gamma_k x_k^j(t)\\ &- h_k(x_k^i(t)) + \gamma_k x_k^i(t)\big)^2\Big)\\ \leq&\frac{1}{2}\sum_{k=1}^\varsigma p_k\Big(\theta_k\tilde{x}_k(t)^T\Xi LL^T\Xi\tilde{x}_k(t)\\ &- \frac{1}{\theta_k}\sum_{j>i}^\varsigma \beta_k^2 Q_{ij}\big(x_k^j(t) - x_k^i(t)\big)^2\Big)\\ =&\frac{1}{2}\sum_{k=1}^\varsigma p_k\tilde{x}_k(t)^T(\theta_k\Xi LL^T\Xi + \frac{\beta_k^2}{\theta_k}Q)\tilde{x}_k(t).\end{aligned} \tag{11}$$

By combining (10) and (11), therefore, we have

$$\begin{aligned}V_2(t) \leq&\sum_{k=1}^\varsigma p_k\tilde{x}_k(t)^T\big(\delta_k U + \sigma(t)(-\gamma_k\Xi L\\ &+ \frac{\theta_k}{2}\Xi LL^T\Xi + \frac{\beta_k^2}{2\theta_k}Q)\big)\tilde{x}_k(t)\end{aligned}$$

$$+ \sum_{k=\varsigma+1}^{n} p_k \tilde{x}_k(t)^T (\delta_k U - \sigma(t)\gamma_k \Xi L)\tilde{x}_k(t) \tag{12}$$

and thus

$$\begin{aligned} V_3(t) &= (-\sigma + \rho\sigma(t))X(t)^T \mathbf{U_P} X(t) \\ &= (-\sigma + \rho\sigma(t)) \sum_{k=1}^{n} p_k \sum_{i=1}^{N}\sum_{j=1}^{N} x_k^i(t)^T U_{ij} x_k^j(t) \\ &= (-\sigma + \rho\sigma(t)) \sum_{k=1}^{n} p_k \tilde{x}_k(t)^T U \tilde{x}_k(t). \end{aligned} \tag{13}$$

Now substitute inequalities (9), (12) and (13) into (8) and we have

$$\begin{aligned} \dot{V}(t) \leq & - \varepsilon X(t)^T U \otimes I X(t) \\ & + \sum_{k=1}^{n} p_k \tilde{x}_k(t)^T (\delta_k - \sigma) U \tilde{x}_k(t) \\ & + \sigma(t) \sum_{k=1}^{\varsigma} p_k \tilde{x}_k(t)^T (\rho U - \gamma_k \Xi L \\ & + \frac{\theta_k}{2} \Xi L L^T \Xi + \frac{\beta_k^2}{2\theta_k} Q) \tilde{x}_k(t) \\ & + \sigma(t) \sum_{k=\varsigma+1}^{n} p_k \tilde{x}_k(t)^T (\rho U - \gamma_k \Xi L) \tilde{x}_k(t) \end{aligned}$$

so that by choosing σ and ρ such that $\delta_k - \sigma \leq 0$, $\rho U - \gamma_k \Xi L \leq 0$ and $\rho U - \gamma_k \Xi L + \frac{\theta_k}{2} \Xi L L^T \Xi + \frac{\beta_k^2}{2\theta_k} Q \leq 0$ in inequality (5), we have

$$\frac{dV(t)}{dt} < 0.$$

Therefore, we have $x_i(t) \to x^\xi(t)$ and $\dot{\sigma}(t) \to 0$ so that, by the Cauchy convergence principle, $\sigma(t)$ converges to some final coupling strength σ_0. The nonlinear coupling system (3) thus attains global synchronization. ■

Remark 1: The final coupling strength can therefore be reduced by making a suitable choice for the value of α, as is illustrated in the following numerical simulation.

IV. Numerical Simulation

In this section, we present some numerical simulation results that verify the theorem given in the previous section. In order to make a fair comparison with the results of [20], we used the same data set for the simulations.

Consider a network with $N = 4$ business entities and a behavior function that is described by the Lorenz oscillator

$$\begin{cases} \dot{x}_1^i = a(x_2^i - x_1^i) \\ \dot{x}_2^i = b x_1^i - x_2^i + x_1^i x_3^i \quad i = 1, 2, 3, 4, \\ \dot{x}_3^i = x_1^i x_2^i - c x_3^i \end{cases} \tag{14}$$

where $a = 10$, $b = 28$, $c = 8/3$.

By referring to the asymmetrical coupling matrix $L = [l_{ij}]$, we take

$$L = \begin{bmatrix} 1.7058 & -0.5913 & -0.0195 & -1.0950 \\ -0.6145 & 2.5367 & -0.0482 & -1.8740 \\ -0.5077 & -0.3803 & 1.3163 & -0.4283 \\ -1.6924 & -1.0091 & -0.3179 & 3.0194 \end{bmatrix} \tag{15}$$

whose left eigenvector corresponding to the eigenvalue 0 is $\xi = (0.3977, 0.2226, 0.0852, 0.2945)^T$. It is clear that $\xi^T L = 0$.

We take the nonlinear functions $h(x^i(t)) = (h_1(x_1^i(t)), h_2(x_2^i(t))$ and $h_3(x_3^i(t)))^T = (0, 3x_2^i(t) + \sin(x_2^i(t)), 0)^T$ so that $h_2(\cdot) \in NCF(3,1)$, $h_1(\cdot) = h_3(\cdot) = 0 \in NCF(0,0)$ and choose the initial values

$$x^1(0) = (98.6337, -51.8635, 32.7368)^T,$$
$$x^2(0) = (23.4057, 2.1466, -100.3944)^T,$$
$$x^3(0) = (-98.6337, 51.8635, -32.7368)^T,$$
$$x^4(0) = (-23.4057, -2.1466, 100.3944)^T.$$

The errors between the nodes (which is a measure of synchronization) are then defined by

$$E(t) = \sqrt{\sum_{i>j} (x^i(t) - x^j(t))^T (x^i(t) - x^j(t))/4}.$$

Calculations then show that, by taking $\theta_2 = 3$, the eigenvalues of the matrix $-3\Xi L + 3\Xi L L^T \Xi/2 + Q/6$ are: 0, -0.2363, -1.2927, -1.1493 and $P = \text{diag}\{1, 1, 1\}$.

Fig. 1(a) and Fig. 1(b) show the evolution of $E(t)$ and $\sigma(t)$ when the Lorenz oscillators are coupled by L with final coupling strength $\sigma_0 = 0.6068$ for $\alpha = 0.00006$. Fig (1(c)) shows the evolution of $E(t)$ when the Lorenz oscillators are coupled by L with fixed coupling strength $\sigma_0 = 0.6068$.

Fig. 2(a) and Fig. 2(b) show the evolution of $E(t)$ and $\sigma(t)$ when the Lorenz oscillators are coupled by L with final coupling strength $\sigma_0 = 0.4844$ for $\alpha = 0.0001$.

Fig. 3 shows the relationship between α and σ_0 when the Lorenz oscillators are coupled by L. As the final coupling strength is very small, when compared with the coupling strength of the [20], the method of adaptive coupling strengths is better for practical applications.

V. Conclusion

In this paper, we study the synchronization problem for a class of coupled supply chain complex networks with an asymmetrical coupling matrix and nonlinear coupling functions. By constructing a suitable adaptive controller, we derive a sufficient condition for global synchronization in which all the business entities can operate harmoniously with stable manufacturing and consumer factors. Numerical examples then demonstrate the effectiveness of the theoretical results.

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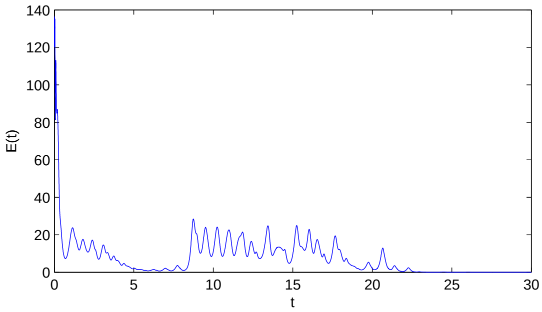
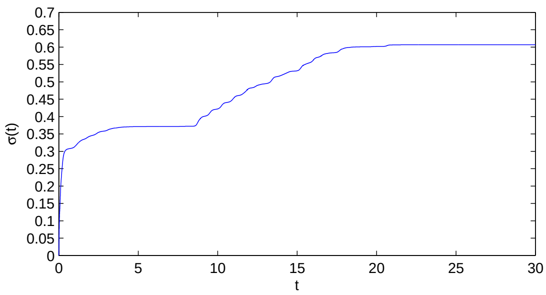
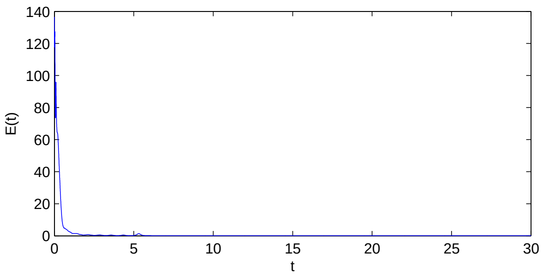


(a) The error E(t) with $\alpha = 0.00006$.

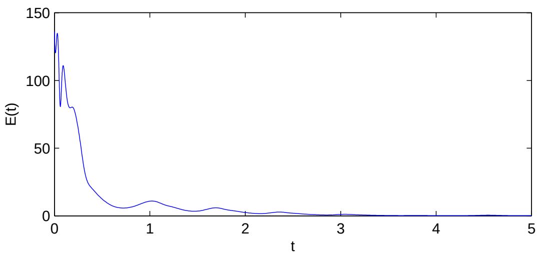


(b) The adaptive coupling strength of $\sigma(t)$ for $\alpha = 0.00006$.

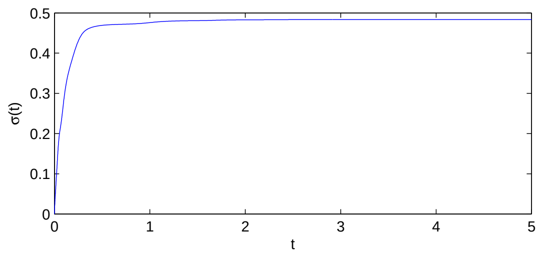


(c) $\sigma_0 = 0.6068$.

Figure 1. The synchronization of system (1).



(a) The error $E(t)$ with $\alpha = 0.0001$.



(b) The adaptive coupling strength with $\sigma(t)$ with $\alpha = 0.0001$.

Figure 2. The synchronization of system (1).

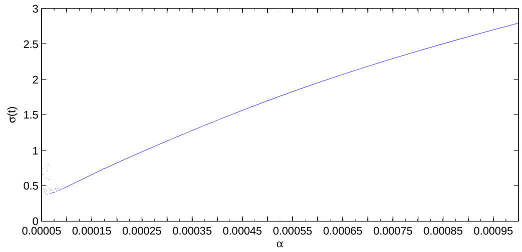


Figure 3. The relationship of between α and σ_0.

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