Adaptive Actuator Failure Compensation of Uncertain Nonlinear Systems with Dead-Zone Inputs

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***Abstract*—As is well known, dead-zone nonlinearity often exists in practical actuators and unknown actuator failures may occur during system operation. However, available results based on adaptive approaches to compensate unknown failures of such actuators are still very limited. In this paper, we address such a problem by considering a class of uncertain nonlinear systems with dead-zone hysteresis nonlinearities inputs. An adaptive control scheme is proposed by using backstepping technology to compensate the uncertainties caused by unknown failures of dead-zone actuators. The boundedness of all signals is established for the closed loop system and the desired output tracking performance is maintained for any unknown actuator failure.**

***Index Terms*—Nonlinear systems, Dead-zone, Adaptive control, Hysteresis**

I. INTRODUCTION

As we all know, hysteresis nonlinearity exists in many physical actuators such as dead-zone, backlash, saturation, and so on. Furthermore, these non-smooth nonlinearities in such actuators are often unknown in parameters and time-variant. The presence of hysteresis nonlinearity in feedback control systems may cause severe deterioration of the system performance. Thus, more and more researchers have tended to pay more attention to the study of non-smooth nonlinearities in practical systems.

Dead-zone is one of the most important hysteresis in practice. It will severely limit the performances of system. But in order to simplify the design process, it is often ignored in control design and stability analysis. This neglect will affect the performance of closed-loop system inevitably, even lead to the instability. So the designer must face the difficulties caused by dead-zone hysteresis. Several control schemes based on adaptive techniques have been proposed to compensate for dead-zone hysteresis in recent years, for example in [1-6]. The adaptive output feedback control scheme was proposed for a class of uncertain nonlinear systems with non-symmetric dead-zone input. The boundedness of all the signals can be ensured by this output feedback controller. In [4], a DIARC scheme was proposed for a class of uncertain systems with non-symmetrical dead-zone nonlinearity. In [5], by introducing a smooth inverse function of the dead-zone, an output feedback control scheme was developed. The system performance was improved.

On the other hand, fault seems inevitable in practice control systems [6-7]. Such failures which may lead to instability or even catastrophic accidents are often uncertain in time, value and pattern. In recent ten years, several adaptive design methods have been proposed to address the unknown failure compensation see for example [8-12]. Based on prescribed performance bound technology, an adaptive control scheme was developed in [11] and transient performance was guaranteed by this control law. In [12] infinite number of failures and faults are considered and a new adaptive compensation scheme was proposed to result this important issue.

Because actuator failure is inevitable and dead-zone is a inherent characteristic of actuators, so it is important to study the controller design under unknown failure of dead-zone actuators. Recently we have given the adaptive compensation controller design for nonlinear systems with unknown failures of backlash actuators [13-14]. However there is still no result about the compensation for unknown failures of actuators which exhibit dead-zone hysteresis nonlinearity. In this note, we will address such a problem for a class of nonlinear systems with dead-zone hysteresis. Note that unknown actuator failures are uncertain in patterns, values and time. Thus the designed control signals for the actuators should accommodate such uncertainties in addition to system parametric uncertainties. Also they should be able to compensate for failure and dead-zone hysteresis effects of the actuators and maintain system stability as well as tracking performance. An adaptive control scheme is proposed in this paper. By introducing the smooth function $tanh_a(\chi)$ in the controller design, the possible chattering caused by traditional control scheme was avoided. With the properties of function $tanh_a(\chi)$, the boundedness of all closed-loop system signals are guaranteed and the desired output tracking performance can be ensured too.

II. PROBLEM STATEMENT

The following nonlinear systems with uncertain parameters is considered

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$$\begin{aligned}&\dot{x}_1 = x_2 + \varphi_1^T(x_1)\theta\\ &\quad\vdots\\ &\dot{x}_n = \varphi_0(x) + \varphi_n^T(x)\theta + \sum_{i=1}^{m} b_i u_i + d_1(t)\\ &y = e_1^T x\end{aligned} \tag{1}$$

where $x = [x_1, x_2, \cdots, x_n]$ are system states, $y \in R$ is output and $u_i \in R$ is input. $\varphi_0(x) \in R$ and $\varphi_i \in R^p$ are known functions. $e_1 = [1, 0, \cdots, 0]^T$ is an unit vector. $\theta \in R^p$ and $b_i \in R$ are unknown constants. $d_1(t)$ is bounded disturbance.

We now consider the i th hysteretic actuator which may fail during its operation. It exhibits dead-zone characteristic behavior denoted as $v_i = D_i(u_{ci})$ with v_i as output and u_{ci} being input. The mathematical model of dead-zone described by

$$v_i = \begin{cases} m_i(u_{ci} - b_{ri}) & u_{ci} \geq b_{ri} \\ 0 & b_{ri} < u_{ci} < b_{li} \\ m_i(u_{ci} - b_{li}) & u_{ci} \leq b_{li} \end{cases} \tag{2}$$

where $b_{ri} \geq 0, b_{li} \leq 0$ and $m_i > 0$ are unknown constants. According to the analysis in [1], we can get

$$v_i(t) = m_i u_{ci}(t) + \bar{d}_i(u_{ci}) \tag{3}$$

$$\bar{d}_i(u_{ci}) = \begin{cases} -mb_{ri} & u_{ci} \geq b_{ri} \\ -mu_{ci} & b_{ri} < u_{ci} < b_{li} \\ -mb_{li} & u_{ci} \leq b_{li} \end{cases}$$

It is clear that $\bar{d}_i$ is bounded as shown in [1]. As in [12]- [14], the failure of the i th actuator at time instant t_{if} can be modeled as follows

$$u_i = \delta_i v_i + \bar{u}_i \qquad (\forall\, t \geq t_{if},\ \delta_i \bar{u}_i = 0) \tag{4}$$

where $\delta_i \in [0,1]$ and $\bar{u}_i$ are unknown constants. The i th actuator is called partial loss of effectiveness when $0 < \delta_i < 1$. If $\delta_i = 0$, it indicates the total loss of effectiveness.

Considering the dead-zone hysteresis and the unknown actuator failures given in (4), system (1) can be rewritten as follows

$$\begin{aligned}&\dot{x}_1 = x_2 + \varphi_1^T(x_1)\theta\\ &\quad\vdots\\ &\dot{x}_n = \varphi_0(x) + \varphi_n^T(x)\theta + \sum_{i=1}^{m} b_i \delta_i m_i u_{ci} + \sum_{i=1}^{m} b_i \bar{u}_i + d(t)\\ &y = e_1^T x\end{aligned} \tag{5}$$

where $d(t) = \sum_{i=1}^{m} \delta_i \bar{d}_i + d_1(t)$. Similar in [13] [14], the term $d(t)$ is bounded.

III. Design of Adaptive Controllers

Our control purpose is to design state feedback adaptive control law to guarantee the stability of closed loop system in the meaning of all signals are bounded. To derive a suitable adaptive control scheme, the following Assumptions are made.

Assumption 1. $b_i \neq 0$ and $sign(b_i)$ is known. Without loss of generality, we let $b_i > 0$.

Assumption 2. Reference signal $y_r(t)$ and it's i -order $(i = 1, 2, \ldots, n-1)$ derivatives are known and bounded.

The following assumption about actuator failure is needed.

Assumption 3. The control objectives can be achieved by the remaining actuation power for any up to $m-1$ actuator failures. Also any actuator can change only from normal to partial failure or total failure once.

Remark 1: As explained in [8], [12] and [13], the above assumption is a basic assumption to ensure the controllability of the closed-loop system with the remaining actuation power. Any actuator fails only once and the amount of actuators is finite. Hence, there exists a finite time instant T_f after which no new failure will occur.

First of all, the function $tanh_a(\chi)$ is defined as follows

$$tanh_a(\chi) = \frac{a^{\chi} - a^{-\chi}}{a^{\chi} + a^{-\chi}} \tag{6}$$

where $a > 1$ is a constant. It is clear that this function is sufficient smooth.

The following changes of coordinates are introduced for using backstepping technology to design adaptive control law.

$$\begin{aligned}&z_1 = x_1 - y_r\\ &z_i = x_i - \alpha_{i-1} - y_r^{(i-1)}, \quad (i = 2, \cdots, n)\end{aligned} \tag{7}$$

where variable z_1 is tracking error and α_{i-1} $(i = 1, 2, \ldots, n)$ is the virtual control in step i .

Step 1: From (4) and (6) the derivative of z_1 is

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_r = z_2 + \alpha_1 + \varphi_1^T(x_1)\theta \tag{8}$$

The virtual control α_1 is designed as

$$\alpha_1 = -c_1 z_1 - \varphi_1^T(x_1)\hat{\theta} \tag{9}$$

where c_1 is a positive constant and $\hat{\theta}$ is an estimate of unknown parameters θ . We define a positive definite Lyapunov function as follows

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \tag{10}$$

where $\tilde{\theta} = \theta - \hat{\theta}$, Γ is a positive definite matrix. From (8) (9) (10) the derivative of V_1 is

$$\dot{V}_1 = -c_1 z_1^2 + z_1 z_2 - \tilde{\theta}^T \Gamma^{-1} (\dot{\hat{\theta}} - \tau_1) \tag{11}$$

where turning function τ_1 is

$$\tau_1 = \Gamma \varphi_1 z_1 \tag{12}$$

Step 2: The derivative of z_2 is

$$\dot{z}_2 = z_3 + \alpha_2 + \varphi_2^T\theta - \frac{\partial \alpha_1}{\partial x_1}(x_2 + \varphi_1^T\theta) - \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}} - \frac{\partial \alpha_1}{\partial y_r} y_r^{(1)} \tag{13}$$

Virtual control α_2 can be chosen as

$$\alpha_2 = -c_2 z_2 - z_1 - (\varphi_2^T - \frac{\partial\alpha_1}{\partial x_1}\varphi_1^T)\hat{\theta} + \frac{\partial\alpha_1}{\partial x_1}x_2 + \frac{\partial\alpha_1}{\partial y_r}y_r^{(1)} + \frac{\partial\alpha_1}{\partial\hat{\theta}}\tau_2 - \frac{\partial\alpha_1}{\partial\hat{\theta}}l_\theta(\hat{\theta}-\theta_0) \tag{14}$$

where c_2, l_θ are positive constants and θ_0 is positive designed constant. The turning function τ_2 is

$$\tau_2 = \tau_1 + \Gamma(\varphi_2 - \frac{\partial\alpha_1}{\partial x_1}\varphi_1)z_2 \tag{15}$$

We consider the following Lyapunov function

$$V_2 = V_1 + \frac{1}{2}z_2^2 \tag{16}$$

Then the derivative of V_2 is

$$\dot{V}_2 = -\sum_{i=1}^{2}c_i z_i^2 + z_2 z_3 - \tilde{\theta}^T\Gamma^{-1}(\dot{\hat{\theta}} - \tau_2) - z_2\frac{\partial\alpha_1}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \tau_2 + l_\theta(\hat{\theta}-\theta_0)) \tag{17}$$

Step $i (i = 3,\ldots,n-1)$: In this step, the following Lyapunov function V_i is considered

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 \tag{18}$$

We choose the virtual control as

$$\begin{aligned}\alpha_i = &-c_i z_i - z_{i-1} - (\varphi_i^T - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\varphi_k^T)\hat{\theta} \\ &+ \sum_{k=1}^{i-1}(\frac{\partial\alpha_{i-1}}{\partial x_k}x_{k+1} + \frac{\partial\alpha_{i-1}}{\partial y_r^{(k-1)}}y_r^{(k)}) \\ &+ \sum_{k=2}^{i-1}\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma(\varphi_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\varphi_k)z_k \\ &+ \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\tau_i - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}l_\theta(\hat{\theta}-\theta_0)\end{aligned} \tag{19}$$

where c_i is a positive constant and turning function τ_i is

$$\tau_i = \tau_{i-1} + \Gamma(\varphi_i - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}\varphi_k)z_i \tag{20}$$

Step n : From (7) , we have

$$z_n = x_n - \alpha_{n-1} - y_r^{(n-1)} \tag{21}$$

And then, the derivative of z_n can be rewritten as

$$\begin{aligned}\dot{z}_n = &\varphi_0 + \varphi_n^T\theta + \sum_{i=1}^{m}b_i(\delta_i m_i u_{ci} + \bar{u}_i) - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}(x_{k+1} + \varphi_k^T\theta) \\ &- \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_r^{(k-1)}}y_r^{(k)} - y_r^{(n)} - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} + d(t)\end{aligned} \tag{22}$$

The adaptive control law and parameters update laws are designed as following.

Control law:

$$u_{ci} = \hat{\mu}^T\omega \tag{23}$$

where $\hat{\mu}$ is the estimate of constant vector $\mu = (\mu_1, \mu_{21}, \cdots, \mu_{2m})^T$ which value is based on the failure pattern. $\omega = (\bar{\alpha}, 1, \cdots, 1)^T$ is a known vector and has the same dimension with μ . $\bar{\alpha}$ is the virtual control in this step

$$\begin{aligned}\bar{\alpha} = &-c_n z_n - z_{n-1} - \varphi_0 - (\varphi_n^T - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}\varphi_k^T)\hat{\theta} \\ &+ \sum_{k=1}^{n-1}(\frac{\partial\alpha_{n-1}}{\partial x_k}x_{k+1} + \frac{\partial\alpha_{n-1}}{\partial y_r^{(k-1)}}y_r^{(k)}) + \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\tau_n \\ &+ \sum_{k=2}^{n-1}z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma(\varphi_n - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}\varphi_k) \\ &- \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}l_\theta(\hat{\theta}-\theta_0) + y_r^{(n)} - tanh_a(\frac{z_n}{\varepsilon})\hat{D}\end{aligned} \tag{24}$$

Update laws:

$$\begin{aligned}&\dot{\hat{D}} = \eta z_n tanh_a(\frac{z_n}{\varepsilon}) - \eta l_D(\hat{D} - D_0) \\ &\dot{\hat{\theta}} = \tau_n - l_\theta\Gamma(\hat{\theta}-\theta_0); \dot{\hat{\mu}} = -\Gamma_\mu z_n\omega - l_\mu\Gamma_\mu(\mu - \mu_0)\end{aligned} \tag{25}$$

where ε is a positive constant and Γ_μ is a positive matrix. $\hat{D}$ are estimations of D and D is the upper bound of disturbance. η, l_D, l_μ , D_0 are positive constants and μ_0 is positive vector. Turning function τ_n is designed as

$$\tau_n = \tau_{n-1} + \Gamma(\varphi_n - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}\varphi_k)z_n \tag{26}$$

Remark 2: Noting that if $D_0 = \theta_0 = 0$ and $\mu_0 = 0$, the control scheme is just the traditional control scheme. These constants introduced in control scheme will be used to discuss the boundedness of all signals of closed-loop system.

Suppose that p_j actuators are faulty and no new normal actuator fails in time interval (T_j, T_{j+1}) , $(j = 0, 1, \cdots, f)$. Let the set Q_{jT} denotes the actuators of total failure in interval (T_j, T_{j+1}) . Now consider the following Lyapunov function in time interval (T_j, T_{j+1})

$$V_{nj} = V_{n-1} + \frac{1}{2}z_n^2 + \sum_{i\in\bar{Q}_{jT}}\frac{b_i\delta_i m_i}{2}\tilde{\mu}^T\Gamma_\mu^{-1}\tilde{\mu} + \frac{1}{2\eta}\tilde{D}^2 \tag{27}$$

where $\tilde{\mu} = \mu - \hat{\mu}$ and $\tilde{D} = D - \hat{D}$. We choose μ as

$$\begin{aligned}&\mu_1 = (\sum_{i\in\bar{Q}_{jT}}b_i\delta_i m_i)^{-1}; \quad \mu_{2i} = 0, (i \in \bar{Q}_{jT}) \\ &\mu_{2i} = b_i\bar{u}_i(-\sum_{i\in\bar{Q}_{jT}}b_i\delta_i m_i)^{-1}, (i \in Q_{jT})\end{aligned} \tag{28}$$

It is clear that $\sum_{i\in\bar{Q}_{jT}}\delta_i b_i m_i\mu^T\omega = \bar{\alpha} - \sum_{i\in Q_{jT}}b_i\bar{u}_i$. With (27), the derivative of V_{nj} is

$$\begin{aligned}\dot{V}_{nj} &= -\sum_{k=1}^{n} c_k z_k^2 + z_n(\varphi_n^T - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}\varphi_k^T)\tilde{\theta} \\ &- z_n\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \tau_n + l_\theta(\hat{\theta}-\theta_0)) + z_n d(t) \\ &- \sum_{k=2}^{n-1} z_k \frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \tau_{n-1} + l_\theta(\hat{\theta}-\theta_0)) - \frac{1}{\eta}\tilde{D}\dot{\hat{D}} \\ &+ z_n\sum_{k=2}^{n-1} z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma(\varphi_n - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}\varphi_k) \\ &- z_n(tanh_a(\frac{z_n}{\varepsilon})\hat{D}) - z_n\sum_{i\in\bar{Q}_{jT}}\delta_i b_i m_i\tilde{\mu}^T\omega \\ &- \sum_{i\in\bar{Q}_{jT}} b_i\delta_i m_i\tilde{\mu}^T\Gamma_\mu^{-1}\dot{\hat{\mu}} - \tilde{\theta}^T\Gamma^{-1}(\dot{\hat{\theta}} - \tau_{n-1})\end{aligned} \tag{29}$$

The derivation of V_{nj} can be rewritten as

$$\begin{aligned}\dot{V}_{nj} &= -\sum_{k=2}^{n} z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \tau_n + l_\theta(\hat{\theta}-\theta_0)) + z_n d(t) \\ &- \sum_{k=1}^{n} c_k z_k^2 - \tilde{\theta}^T\Gamma^{-1}(\dot{\hat{\theta}} - \tau_n) - \frac{1}{\eta}\tilde{D}\dot{\hat{D}} \\ &- \sum_{i\in\bar{Q}_{jT}} b_i\delta_i m_i\tilde{\mu}^T\Gamma_\mu^{-1}(\dot{\hat{\mu}} + \Gamma_\mu z_n\omega) \\ &- z_n(tanh_a(\frac{z_n}{\varepsilon})\hat{D})\end{aligned} \tag{30}$$

Lemma 1: To $tanh_a(\cdot)$ function the following inequality can be achieved

$$|\chi| - \chi tanh_a(\frac{\chi}{\varepsilon}) \le \frac{k\varepsilon}{lna}, \forall\chi\in R, \varepsilon > 0 \tag{31}$$

where $k = 0.2785$.

Proof: From (6) we can get

$$tanh_a(\chi) = \frac{a^\chi - a^{-\chi}}{a^\chi + a^{-\chi}} = \frac{e^{\ln a\chi} - e^{-\ln a\chi}}{e^{\ln a\chi} + e^{-\ln a\chi}} = tanh(\ln a\chi) \tag{32}$$

So we have

$$|\chi| - \chi tanh_a(\frac{\chi}{\varepsilon}) = |\chi| - \chi tanh(\frac{\chi}{(\varepsilon/\ln a)}) \le \frac{k\varepsilon}{lna} \quad \square$$

With Lemma 1, the derivative of V_{nj} can be rewritten as

$$\begin{aligned}\dot{V}_{nj} &= -\sum_{k=1}^{n} c_k z_k^2 - \tilde{\theta}^T\Gamma^{-1}(\dot{\hat{\theta}} - \tau_n) - \sum_{k=2}^{n} z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}(\dot{\hat{\theta}} \\ &- \tau_n + l_\theta(\hat{\theta}-\theta_0)) + |z_n| D - z_n(tanh_a(\frac{z_n}{\varepsilon})\hat{D}) \\ &- \sum_{i\in\bar{Q}_{jT}} b_i\delta_i m_i\tilde{\mu}^T\Gamma_\mu^{-1}(\dot{\hat{\mu}} + \Gamma_\mu z_n\omega) - \frac{1}{\eta}\tilde{D}\dot{\hat{D}} \\ &\le -\sum_{k=1}^{n} c_k z_k^2 - \tilde{\theta}^T\Gamma^{-1}(\dot{\hat{\theta}} - \tau_n) - \sum_{k=2}^{n} z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \tau_n \\ &+ l_\theta(\hat{\theta}-\theta_0)) - \sum_{i\in\bar{Q}_{jT}} b_i\delta_i m_i\tilde{\mu}^T\Gamma_\mu^{-1}(\dot{\hat{\mu}} + \Gamma_\mu z_n\omega) \\ &- \frac{1}{\eta}\tilde{D}(\dot{\hat{D}} - \eta z_n tanh_a(\frac{z_n}{\varepsilon})) + \frac{\varepsilon}{lna}D\end{aligned} \tag{33}$$

With the update laws (25), we have

$$\begin{aligned}\dot{V}_{nj} &\le -\sum_{k=1}^{n} c_k z_k^2 + \frac{\varepsilon}{lna}D + \tilde{\theta}^T l_\theta(\hat{\theta}-\theta_0) + \tilde{D}l_D \\ &(\hat{D} - D_0) + \sum_{i\in\bar{Q}_{jT}} b_i\delta_i m_i\tilde{\mu}^T l_\mu(\hat{\mu}-\mu_0)\end{aligned} \tag{34}$$

Noting that the following inequalities

$$\begin{aligned} l_D\tilde{D}(\hat{D}-D_0) &\le -\frac{1}{2}l_D\tilde{D}^2 + \frac{1}{2}l_D(D-D_0)^2 \\ l_\theta\tilde{\theta}^T(\hat{\theta}-\theta_0) &\le -\frac{1}{2}l_\theta\|\tilde{\theta}\|^2 + \frac{1}{2}l_\theta\|\theta-\theta_0\|^2 \\ l_\mu\tilde{\mu}^T(\hat{\mu}-\mu_0) &\le -\frac{1}{2}l_\mu\|\tilde{\mu}\|^2 + \frac{1}{2}l_\mu\|(\mu-\mu_0)\|^2\end{aligned} \tag{35}$$

Then

$$\dot{V}_{nj} \le -\sum_{k=1}^{n} c_k z_k^2 - \frac{1}{2}\sum_{i\in\bar{Q}_{jT}} b_i\delta_i m_i l_\mu\|\tilde{\mu}\|^2 - \frac{l_\theta}{2}\|\tilde{\theta}\|^2 - \frac{l_D}{2}\tilde{D}^2 + \Xi_j \tag{36}$$

where

$$\begin{aligned}\Xi_j &= \frac{1}{2}l_\theta\|\theta-\theta_0\|^2 + \sum_{i\in\bar{Q}_{jT}} b_i\delta_i m_i\frac{1}{2}l_\mu\|(\mu-\mu_0)\|^2 \\ &+ \frac{1}{2}l_D(D-D_0)^2 + \frac{\varepsilon}{lna}D\end{aligned} \tag{37}$$

Because θ , μ , D , θ_0 , μ_0 , D_0 are constants and $l_\theta, l_\mu, l_D, lna, \varepsilon, \sum_{i\in\bar{Q}_{jT}} b_i\delta_i m_i$ are positive designed constants, Ξ_j is positive and bounded.

Remark 3: The value of Ξ_j is based on the failure such as values of $\bar{Q}_{jT}$, δ_i and μ in time interval (T_j, T_{j+1}) . Because in (T_j, T_{j+1}) all above parameters are fixed, Ξ_j is constant in this interval.

Theorem 1: Considering the system (1) with the m hysteresis inputs modeled in (2) and unknown failures described by (4), under the control law (23) and update laws (25), all signals in close-loop system are bounded. In addition, the following tracking performance is achieved, i.e.

$$lim_{t\to\infty}|y - y_r| \le \sqrt{\frac{\Xi_f}{c_1}} \tag{38}$$

Proof: In time interval (T_j, T_{j+1}) , From (27) and (36) we can get the following inequalities

$$\begin{aligned}\dot{V}_{nj} &\le -\min\{c_k, \frac{l_\mu}{2}\sum_{i\in\bar{Q}_{jT}} b_i\delta_i m_i, \frac{l_\theta}{2}, \frac{l_D}{2}\}(\sum_{k=1}^{n} z_k^2 + \|\tilde{\mu}\|^2 + \|\tilde{\theta}\|^2 \\ &+ \tilde{D}^2) + \Xi_j\end{aligned} \tag{39}$$

and

$$\begin{aligned} V_{nj} &\le \frac{1}{2}\sum_{k=1}^{n} z_k^2 + \frac{1}{2}\tilde{\theta}^T\Gamma^{-1}\tilde{\theta} + \sum_{i\in\bar{Q}_{jT}}\frac{b_i\delta_i m_i}{2}\tilde{\mu}^T\Gamma_\mu^{-1}\tilde{\mu} + \frac{1}{2\eta}\tilde{D}^2 \\ &\le \max\{\frac{1}{2}, \frac{\lambda_{\max\Gamma_\mu^{-1}}}{2}\sum_{i\in\bar{Q}_{jT}} b_i\delta_i m_i, \frac{\lambda_{\max\Gamma^{-1}}}{2}, \frac{1}{2\eta}\}(\sum_{k=1}^{n} z_k^2 \\ &+ \|\tilde{\mu}\|^2 + \|\tilde{\theta}\|^2 + \tilde{D}^2)\end{aligned} \tag{40}$$

where $\lambda_{\max\Gamma_\mu^{-1}}$ and $\lambda_{\max\Gamma^{-1}}$ are the maximum eigenvalues of Γ_μ^{-1}, Γ^{-1} From (39) and (40), we have

$$\dot{V}_{nj} \le -\Theta_j V_{nj} + \Xi_j \tag{41}$$

where

$$\Theta_j = \frac{\min\{c_k, \frac{l_\mu}{2}\sum_{i\in\bar{Q}_{jT}} b_i\delta_i m_i, \frac{l_\theta}{2}, \frac{l_D}{2}\}}{\max\{\frac{1}{2}, \frac{\lambda_{\max\Gamma_\mu^{-1}}}{2}\sum_{i\in\bar{Q}_{jT}} b_i\delta_i m_i, \frac{\lambda_{\max\Gamma^{-1}}}{2}, \frac{1}{2\eta}\}}$$

It is clear that Θ_j is a positive constant in time interval (T_j, T_{j+1}). Then we can get

$$V_{nj}(t) \le (V_{nj}(T_j) - \frac{\Xi_j}{\Theta_j})e^{-\Theta_j(t-T_j)} + \frac{\Xi_j}{\Theta_j}, \quad \forall t \in (T_j, T_{j+1}) \tag{42}$$

From (27), the difference between V_{nj} and $V_{n(j-1)}$ is only the coefficients in front of the term $\tilde{\mu}^T\Gamma_\mu^{-1}\tilde{\mu}$. Since the possible jumping of μ is bounded at time instant T_j, we can get

$$\Pi_j = V_{nj}(T_j) - V_{n(j-1)}(T_j)$$
$$= (\sum_{i\in\bar{Q}_{jT}} \frac{b_i\delta_i m_i}{2} - \sum_{i\in\bar{Q}_{(j-1)T}} \frac{b_i\delta_i m_i}{2})\tilde{\mu}^T\Gamma_\mu^{-1}\tilde{\mu} + \sum_{i\in\bar{Q}_{jT}} \frac{b_i\delta_i m_i}{2}\Delta_{j\tilde{\mu}}^T\Gamma_\mu^{-1}\Delta_{j\tilde{\mu}}$$

is bounded, where $\Delta_{j\tilde{\mu}}$ is the jumping of μ. So from $V_{n1}(0)$ is constant and (41), we can get $V_{n1}(T_1)$ is bounded and $V_{n1}(t)$ is bounded for any $t \in (0, T_1)$. Namely all signals are bounded in this time interval. Then $V_{n2}(T_1)$, $V_{n2}(T_2)$ are bounded. From (42) $V_{n2}(t)$ is bounded $\forall t \in (T_1, T_2)$. Then all signals are bounded in time interval (T_1, T_2). So by using the same argument as above we can ensure all signals are bounded in (T_j, T_{j+1}), further we can get in $(0, +\infty)$ all signals are bounded.

From (36), we can get

$$\dot{V}_{nf} \le -\sum_{k=1}^{n} c_k z_k^2 + \Xi_f$$
$$\le -c_1|z_1|^2 + \Xi_f, \quad t \in (T_f, +\infty) \tag{43}$$

By applying the Lasalle-Yoshizzawa theorem, it follows also that $lim_{t\to\infty}|y - y_r| \le \sqrt{\frac{\Xi_f}{c_1}}$ □

IV. Simulation

In this section, we use the aforementioned methodology on a simple system. It can be described as follows

$$\dot{x} = \varphi(x)^T\theta + b_1u_1 + b_2u_2 \tag{44}$$

where u_1, u_2 are the control signals of system and exhibit dead-zone nonlinearity. The known function $\varphi(x)$ is

$$\varphi(x) = \frac{1-e^{-x}}{1+e^{-x}}$$

The actual values of parameters are $\theta = 2$ and $b_1 = b_2 = 1$. Reference signal is $12.5sin(2.3t)$. The unknown parameters in dead-zone are $m_i = 1, b_{ri} = b_{li}$ $(i = 1, 2)$ .In simulation we choose $a = e$ which indicates $tanh_a(\cdot) = tanh(\cdot)$. Feedback gain $c = 30$, $\Gamma_\mu = 0.2, \Gamma = 0.2$, $\eta = 0.2$ and $\varepsilon = 0.1$. The design parameters are chosen as $l_D = l_\theta = l_\mu = 0.1$ and $D_0 = 3$, $\theta_0 = 0.9$, $\mu_0 = 0$. Initial value are chosen as follows: $z(0) = 0.5$, $u_1(0) = u_2(0) = 0$, $\hat{\theta}(0) = 0$, $\hat{D}(0) = 0$, $\mu(0) = 0$.

Figs.1, Figs.2 and Figs.3 are tracking error and input $u_1(t), u_2(t)$ when the actuator $u_2(t)$ is stuck at an unknown value 20 at $t = 1.6$ second. Figs.3, Figs.4 and Figs.5 are tracking error and input $u_1(t), u_2(t)$ when all actuators works normally. Clearly the proposed scheme has been verified effective by these simulation results.

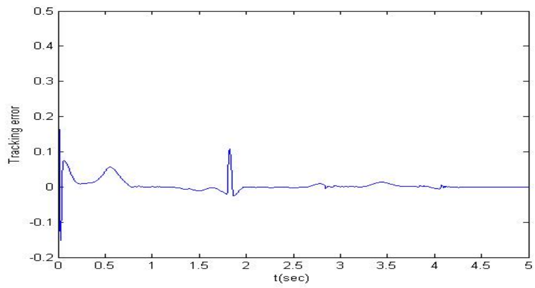

Figure 1. Tracking error (failure)

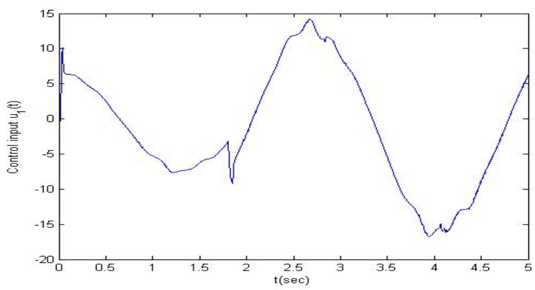

Figure 2. Dead-zone input u1 (failure)

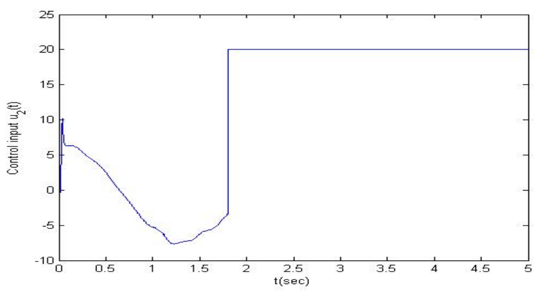

Figure 3. Dead-zone input u2 (failure)

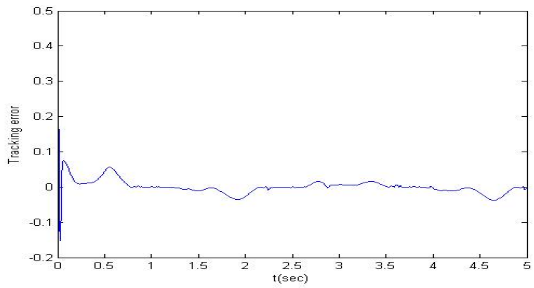

Figure 4. Tracking error (no failure)

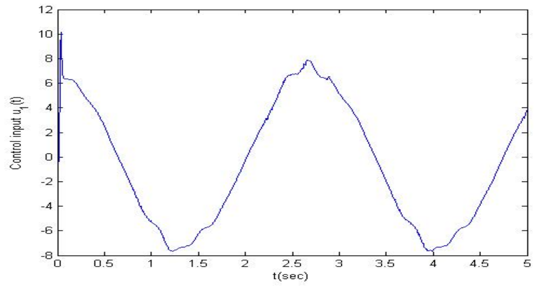

Figure 5. Dead-zone input u1 (no failure)

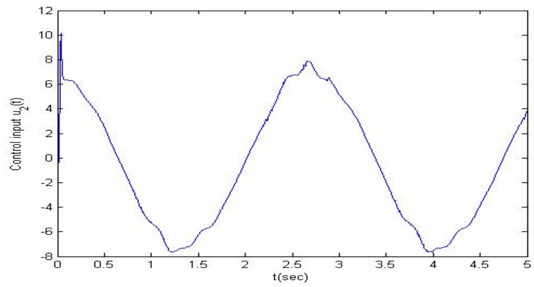

Figure 6. Dead-zone input u2 (no failure)

V. CONCLUSION

A new state feedback control scheme is proposed by using backstepping technology for a class of nonlinear systems preceded by unknown dead-zone hysteresis nonlinearities. The stability in the meaning of all signals being bounded system and desired output tracking performance can be guaranteed by this control law and parameters update laws. Finally Simulation results also illustrate the effectiveness of the control scheme.

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