# Projection onto Convex Sets Method in Spacefrequency Domain for Super Resolution

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*Abstract*—The aim of super resolution is to get high resolution (HR) images/videos from low-resolution (LR) images/videos. The obtained HR images/videos are expected to be clear and have less artifacts. Projection Onto Convex Sets (POCS) in the space domain is a super resolution method. It results in edge oscillation and produces many artifacts. This paper introduces a POCS method in both the space and the frequency domain. Firstly, a frequency domain POCS method is proposed. Then it is combined with the space POCS and the space-frequency POCS is obtained. Compared with the common bilinear interpolation method and the existing POCS method in the space domain, our method may decrease the edge oscillation phenomena and raise the Peak Signal to Noise Ratio (PSNR).

*Index Terms*—super-resolution, Projection Onto Convex Sets (POCS), space-frequency domain, edge oscillation, Peak Signal to Noise Ratio (PSNR)

## I. INTRODUCTION

It is easy for people to get digital images and videos from various channels in today's society. However, the existing resolutions of these digital elements may not meet the requirements of people usually. One way to raise the resolutions is to update the hard devices. It often requires a high cost. The other way is to use software. It creates high resolution (HR) images/videos from Low resolution (LR) ones. This process is called superresolution. It is widely used in the fields of intelligent surveillance [1, 2], remote sensing [3], medical technology [4], mobile devices [5], and so on.

There are a lot of super resolution methods. These methods are mainly divided into two classes [6]. The first is based on learning. In 2002, Reference [7] uses a nearest-neighbor search in the training set to get a one-

pass super resolution algorithm. Reference [8] uses contourlet transformation for video super-resolution. In 2010, Reference [9] combines the partial differential equations' regularization with the learning-based superresolution process. Reference [10] uses nonlinear mappings to coherent features and recognizes the faces in the LR images. In the train set, Reference [11] uses the kernel partial least square to describe the relationship between the LR and the HR images. The second is based on reconstruction. The HR images are reconstructed as the inverses of the LR images in these methods, such as interpolation [12-15], Projection onto Convex Sets (POCS) [16-18], Maximum a Posteriori [19-22], Iterative Back Projection [23-25], and their combination [26].

In this paper, we focus on the conventional POCS method in the space domain. It uses a priori knowledge, and is easy for implementing [27]. However, its reconstructed images may have edge oscillations. We will give a space-frequency POCS instead. By using our method, the edge oscillation phenomena will be decreased, and the Peak Signal to Noise Ratio (PSNR) will be increased.

The rest of the paper is divided into 4 parts. In the next section, the conventional POCS is introduced shortly. Section 3 and Section 4 shows our method for images and videos, respectively. The final conclusions are drawn in the last section.

#### II. POCS METHOD

In this section, we will overview the conventional POCS method [16] shortly.

The principle of the POCS method is simple and direct to view. It uses a priori knowledge of the spatial response characteristics of the imaging system. In the process of reconstructing HR image, there are many constraints. All the constraints, regarded as the boundaries, build a set. The set is made up of some convex sets with good

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properties. This is the solution space of the HR images. Then the required HR image is the optimal solution in the space, that is, the projections onto the convex sets. So the main idea of POCS is to solve a constrained optimization problem.

See Fig. 1. There should be the minimal residuals between the given images and the LR images degenerated by the required HR image. This is the target function in the constrained optimization problem.

The constraints are described as that if the residuals are all less than a given threshold, then the projections from the HR image to the LR ones are obtained; otherwise, the degenerating model is needed to be modified. Both the target function and the constraints are described in the space domain. So we call it space POCS in this paper.



Figure 1. Explanation of POCS method.

### **III. SPACE-FREQUENCY POCS FOR IMAGES**

In this section, we use our space-frequency POCS for image super resolution. The first subsection deals with the image matching process. In the second subsection, our POCS in the frequency domain is introduced. The third subsection combines the space and the frequency domain and gets our space-frequency POCS. We also give examples to show visual comparisons among the bilinear interpolation, the space POCS and our spacefrequency POCS. In the last subsection, we give the PSNR as a quantitative comparison of the above mentioned three methods.

We firstly use bilinear interpolation, a simple and common method, to get a HR reference image, noted as f(x, y), where x and y is corresponding to the row and the column number of pixels, respectively.

### A. Image Matching

There are many images matching method. In this paper, we hope to effectively describe the rotation, translation and scaling movements between two images caused by relative motion between the lens and the scene. Moreover, the method is expected to be used in video cases easily. So we choose the least square image matching based on affine models [28].

Given a reference image  $I_0(x, y)$ , the matched image

 $I_1(x, y)$ , can be presented thought the affine relations as follows.

$$I_{1}(x,y) = I_{0}(x_{0},y_{0}) = I_{0}(x + (a_{x} + b_{x}x + c_{x}y), y + (a_{y} + b_{y}x + c_{y}y))$$
(1)

where  $a_x, b_x, c_x, a_y, b_y, c_y$  are the parameters of the affine transformation. Then the square error of the two images is

$$E(a_x, b_x, c_x, a_y, b_y, c_y) = \sum_{(x,y)} \left[ I_1(x, y) - I_0(x_0, y_0) \right]^2.$$
(2)

Least square matching of the two images is the process of solving the parameters for the minimal E. That is,

$$\underset{a_{x},b_{x},c_{x},a_{y},b_{y},c_{y}}{\arg\min} = E(a_{x},b_{x},c_{x},a_{y},b_{y},c_{y}) .$$
(3)

In the neighborhood of the pixel (x, y), we use the Taylor expansion to approximate  $I_0(x_0, y_0)$ .

$$I_{0}(x_{0}, y_{0}) \approx I_{0}(x, y) -(a_{x} + b_{x}x + c_{x}y)d_{x}(x, y) + (a_{y} + b_{y}x + c_{y}y)d_{y}(x, y),$$
(4)

where

+

$$d_{x}(x,y) = \frac{\partial I_{0}(x,y)}{\partial x} = \frac{1}{2} [I_{0}(x+1,y) - I_{0}(x-1,y)]$$
(5)

and

$$d_{y}(x,y) = \frac{\partial I_{0}(x,y)}{\partial y} = \frac{1}{2} [I_{0}(x,y+1) - I_{0}(x,y-1)]$$
(6)

separately denotes the gradient of  $I_0(x, y)$  along the x and y direction.

Let

$$\Delta I(x, y) = I_1(x, y) - I_0(x, y) .$$
(7)

Then the problem (3) can be represented as

$$\sum_{(x,y)} 2 \left[ q(x,y)^T p - \Delta I(x,y) \right] q(x,y) = 0$$
 (8)

where

$$q = \begin{bmatrix} d_x & xd_x & yd_x & d_y & xd_y & yd_y \end{bmatrix}^T$$
(9)

is the coefficient matrix and

$$p = \left[a_x b_x c_x a_y b_y c_y\right]^T$$
(10)

is the parameter vector. The iterative process is

$$\begin{bmatrix} 1 & 0 & 0 \\ a_x^{i+1} & b_x^{i+1} & c_x^{i+1} \\ a_y^{i+1} & b_y^{i+1} & c_y^{i+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \Delta a_x^i & 1 + \Delta b_x^i & \Delta c_x^i \\ \Delta a_y^i & \Delta b_y^i & 1 + \Delta c_y^i \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ a_x^i & b_x^i & c_x^i \\ a_y^i & b_y^i & c_y^i \end{bmatrix}.$$
(11)

The Matlab program for calculating the affine parameters can be seen in Appendix A. We use small sampling level here. This estimation applies to the small amplitude of movement, which is enough for our examples in this paper. If more accurate estimation of the successive correction parameters is need, then more iterative steps may be used.

#### B. POCS in Frequency Domain

The conventional POCS is in the space domain. This subsection will give a POCS in the frequency domain. We call it frequency POCS. Different from [17], we focus on the norm in the frequency domain.

We use the Gauss function, a very common degrading function in many optical systems, as the Point Spread Function (PSF).

$$h(x_0, y_0) = C e^{\frac{(x-x_0)^2 + (y-y_0)^2}{2\sigma^2}},$$
 (12)

where  $(x_0, y_0)$  is the spread center and  $\sigma$  denotes the blurring level of the image. *C* is the constant to normalize the function. It fulfills

$$\sum_{(x_{n_0}=-w)}^{x_{n_0}=w} \sum_{y_{-y_0}=-w}^{y_{-y_0}=w} h(x_0, y_0) = 1, \qquad (13)$$

where w gives the radius of the support domain of the PSF.

We assume that  $g_k(i, j)$  denotes the observed *k*th LR images. According to the motion vector field, for each pixel  $g_k(i_0, j_0)$  in  $g_k(i, j)$ , we can get the corresponding coordinate of the point in f(x, y), the reference image. The point is noted as  $f(x_0, y_0)$ . Then the estimated LR image is regarded as the image degenerated from the HR image through the Gauss blurring. That is,

$$\hat{g}_{k}(i_{0},j_{0}) = \sum_{x-x_{0}=-w}^{x-x_{0}=-w} \sum_{y-y_{0}=-w}^{y-y_{0}=-w} h(x_{0},y_{0})f(x_{0},y_{0})$$
(14)

The residual between the estimated and the observed LR images can be given by the following formula.

$$r(x_0, y_0) = g_k(i_0, j_0) - \hat{g}_k(i_0, j_0) \quad . \tag{15}$$

If f(x, y) is the ideal HR images, then the residual r(x, y) is equal to zero. Let  $\delta_0$  be a positive number. It is used as a threshold to construct the iterative function. If the residual r is in the threshold range, then f(x, y) will be unchanged, otherwise, f(x, y) will be modified. If r is too small or too large, then it will be increased or decreased until it is near-zero, i.e. in the  $\delta_0$  determined neighborhood of zero. The iterative process can be presented as follows.

$$P[f(x_0, y_0)] = \begin{cases} f(x_0, y_0) + u(r + \delta_0)h(x_0, y_0) & r < -\delta_0 \\ f(x_0, y_0) & -\delta_0 \le r \le \delta_0 \\ f(x_0, y_0) + u(r - \delta_0)h(x_0, y_0) & r > \delta_0 \end{cases}$$
(16)

where u is a constant to control the convergence speed of the iterative process.

The above three formulas, (14), (15) and (16) are all in the space domain. They are used in the space POCS method. We use the Fourier transformation, a common technique, to change them into the frequency domain. In the frequency POCS, our estimated LR image is presented as follows.

$$F(\hat{g}_{k}(i_{0},j_{0})) = \sum_{x-x_{0}=-w}^{x-x_{0}=-w} \sum_{y-y_{0}=-w}^{y-y_{0}=-w} F(h(x_{0},y_{0})) * F(f(x_{0},y_{0})), \quad (17)$$

where F is the Fourier transformation, and "\*" means the convolution operator. Then our residual in the frequency domain is

$$R = \left\| F(g_k(i_0, j_0)) - F(\hat{g}_k(i_0, j_0)) \right\|.$$
(18)

Analogously, our iterative process in the frequency domain is presented as

$$\left\| P \left[ F \left( f(x_0, y_0) \right) \right] \right\| = \begin{cases} \left\| F \left( f(x_0, y_0) \right) \right\| & R < -\delta \\ + u \left( R + \delta \right) \left\| F \left( f(x_0, y_0) \right) \right\| & |R| \le \delta \\ \left\| F \left( f(x_0, y_0) \right) \right\| & |R| \le \delta \\ + u \left( R - \delta \right) \left\| F \left( f(x_0, y_0) \right) \right\| & R > \delta \end{cases}$$

where  $\delta$  is a threshold.

By now, we have two POCS models. One is in the space domain, and the other is in the frequency domain. Then we will combine them with each other in the next subsection.

## C. Combining Space and Frequency POCS

Assume that the HR image obtained by using the space POCS and the frequency POCS is s(x, y) and f(x, y), respectively. Then we use an interpolation method to get the final HR image.

$$G(x, y) = \alpha(x, y)s(x, y) + (1 - \alpha(x, y))f(x, y), \quad (20)$$

where  $\alpha(x, y)$  is the interpolation function defined by

$$\alpha(x, y) = \begin{cases} 0, & (x, y) \in W \\ 1, & (x, y) \notin W \end{cases},$$
(21)

and W is the edge set of the image which may be obtained by an algorithm of edge detection, for example, Canny detection.

To see the super-resolution effects of our method, we give two examples for images. In this paper, we assume that the pixels' size do not changed in the super resolution process. So, raising resolution means the size of image/video increased. In all the examples of the paper, the original LR images/videos are all with smaller size, and the super-resolution HR images/videos are all with bigger size.

Fig. 2 shows the first example. Fig. 2(a) is the LR image with the resolution  $176 \times 144$ . Fig. 2(b), Fig. 2(c), Fig. 2(e) is the obtained HR image with the resolution  $352 \times 288$  by the bilinear interpolation, the traditional space POCS and our space-frequency POCS, respectively.

It is easy for us to see that Fig. 2(c) and Fig. 2(e) are both clearer than Fig. 2(b). However, the difference between Fig. 2(c) and Fig. 2(e) is not obvious. So we magnify the same region of Fig. 2(c) and Fig. 2(e) to the same 800% times. The region is bounded by a blue box in Fig. 2(c) and Fig. 2(e), separately. The magnified region corresponding to Fig. 2(c), Fig. 2(e) is Fig. 2(d), Fig. 2(f), respectively.

By contrast, we can see that the light spots in Fig. 2(e) are clearer than those in Fig. 2(d).





(a) Original LR image.

(b) Bilinear interpolation method.



(c) Space POCS method.

(d) Magnified region in (c).



(e) Our method.





Fig. 3 gives the second example. Fig. 3(a) is the LR image with the resolution  $176 \times 144$ . Fig. 3(b), Fig. 3(c), Fig. 3(e) is the obtained HR image with the resolution  $352 \times 288$  by the bilinear interpolation, the traditional space POCS and our space-frequency POCS, respectively.

It is easy for us to see that Fig. 3(c) and Fig. 3(e) are both clearer than Fig. 3(b). However, the difference between Fig. 3(c) and Fig. 3(e) is not obvious. So we magnify the same region of Fig. 3(c) and Fig. 3(e) to the same 800% times. The region is bounded by a blue box in Fig. 3(c) and Fig. 3(e), separately. The magnified region corresponding to Fig. 3(c), Fig. 3(e) is Fig. 3(d), Fig. 3(f), respectively.

In Fig. 3(d), we can see many black stripes, especially in the edge of the red region. These stripes are all artifacts. They make the edge seems to be wider and oscillating. Our method decreases the phenomena, seen in Fig. 3(f).

Comparing the conventional space POCS method with our space-frequency POCS method from vision effects, we can get the result that our method may decrease the edge oscillation phenomena and artifacts produced by the conventional POCS method and make the image clearer.





(a) Original LR image.



(d) Magnified region in (c).



(e) Our method.

(f) Magnified region in (e).

Figure 3. Image Ex. 2.

#### D. PSNR

In the last subsection, we show some visual results. In this subsection, a quantitative comparison among the bilinear interpolation, the traditional POCS method and our method will be given. We use PSNR to describe the quality of the super-resolution reconstruction.

$$PSNR = 10\log\left(\frac{255^2}{MSE}\right),$$
 (22)

where MSE is the mean square error defined as

$$MSE = \frac{\sum_{y=1}^{y=n} \sum_{x=1}^{x=m} (f(x,y) - f'(x,y))^2}{m \times n},$$
 (23)

and f(x, y), f'(x, y) is separately the real HR image and the HR image obtained by using some algorithm. *m* and *n* is the horizontal and vertical resolution, respectively.

Consider the examples in the last subsection. Their PSNR by using the bilinear interpolation, the traditional POCS and our method is listed in Table 1. From this table, we can see that in the two examples, the PSNRs of our

TABLE I. COMPARISON OF PSNR

Ex.	PSNR(dB)		
	Bilinear interpolation	Space POCS	Our method
1	28.29	28.99	29.42
2	17.72	18.43	18.73

method are greater than those of the conventional POCS and the bilinear interpolation approach.

## IV. SPACE-FREQUENCE POCS FOR VIDEOS

We can use the similar method for video superresolution. Motion estimation is dealt with in the process of image matching. For any frame in the video, we use the previous frame to solve the affine model, and use the current frame to modify the parameters of the model. The process is shown in Fig. 4.



Figure 4. Explanation of affine model.



(a) LR Frame *i*-1

(d) HR Frame i-1



(b) LR Frame *i* 



(e) HR Frame i

Figure 5. Video Ex. 1.

See examples. Fig. 5 shows the first video example consisted of three neighbor frames. The space resolution of the given LR video is  $176 \times 144$ . After super resolution, the row and column resolutions are both doubled. The HR video produced by our method is with the space resolution  $352 \times 288$ .

Fig. 5(a), Fig. 5(b) and Fig. 5(c) are the given LR frames, where Fig. 5(b) is the current *i*th frame, Fig. 5(a) is the pervious (i-1)th frame, and Fig. 5(c) is the next (i+1)th frame.

Fig. 5(d), Fig. 5(e) and Fig. 5(f) are the HR frames constructed by our space-frequency POCS method. They are corresponding to Fig. 5(a), Fig. 5(b) and Fig. 5(c) in order. So the frame number of Fig. 5(d), Fig. 5(e) and Fig. 5(f) is the same as Fig. 5(a), Fig. 5(b) and Fig. 5(c), i.e. i-1, i, i+1, respectively.

Fig. 6 gives another video example. It shows the nonadjacent video frames. The space resolution of the given LR video and the HR video is also  $176 \times 144$  and  $352 \times 288$ , respectively. The HR video is constructed by using our method.

Fig. 6(a), Fig. 6(b) and Fig. 6(c) are the given LR frames. They are nonadjacent. The frame number of Fig. 6(a), Fig. 6(b) and Fig. 6(c) is assumed as i, j and k, respectively.

Fig. 6(d), Fig. 6(e) and Fig. 6(f) are the HR frames constructed by our space-frequency POCS method. The frame number of Fig. 6(d), Fig. 6(e) and Fig. 6(f) is i, j and k, respectively. This means that Fig. 6(d), Fig. 6(e) and Fig. 6(f) is the HR frame corresponding to Fig. 6(a), Fig. 6(b) and Fig. 6(c), respectively.

(c) LR Frame i+1

(f) HR Frame i+1



(a) LR Frame *i* 



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(b) LR Frame j
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(c) LR Frame k



(d) HR Frame *i* 

(e) HR Frame j

(f) HR Frame k

Figure 6. Video Ex. 2.

#### V. CONCLUSIONS

In this paper, we give a space-frequency POCS method. The method can be used to super resolution for either images or videos. Comparing with the existing space POCS, our method reduces the edge oscillation artifacts and increases the PSNR of images.

However, to solve the constrained optimization problems, some iteration steps are used. So our method is slow in the implementation. We will consider its acceleration in future work.

## APPENDIX A MATLAB CODE FOR AFFINE PARAMETERS

% Affine Parameter Calculation Matlab code function P = Affine(Image1, Image2, Num); % Initialization parameter vector P(1:6) = 0;for Level = 2:-1:0% Get sampling step Step =  $2^{\text{Level}}$ ; P = [P(1) P(2) P(3) P(4) P(5) P(6)];% Compute correlation matrices Im1 = Corr(Image1, Step); Im2 = Corr(Image2, Step); for i=1:3 % Motion compensate Compen = Comp(Im2, P, Num-1); % Calculate spatial-temporal gradients [Gx Gy Gt] = Grad(Im1, Compen);% Calculate increments of parameters InP = InParameter(Gx, Gy, Gt, Num); % Calculate P P = P + InP;end

end

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